# Deformation of surfaces, integrable systems and Self-Dual Yang-Mills equation

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### Abstract

We conjecture that many (maybe all) integrable systems and spin systems in 2+1 dimensions can be obtained from the (2+1)-dimensional Gauss-Mainardi-Codazzi and Gauss-Weingarten equations, respectively. We also show that the (2+1)-dimensional Gauss-Mainardi-Codazzi equation which describes the deformation (motion) of surfaces is the exact reduction of the Yang-Mills-Higgs-Bogomolny and Self-Dual Yang-Mills equations. On the basis of this observation, we suggest that the (2+1)-dimensional Gauss-Mainardi-Codazzi equation is a candidate to be integrable and the associated linear problem (Lax representation) with the spectral parameter is presented.

## 1    Introduction

Several nonlinear phenomena in physics, modeled by the nonlinear differential equations (NDE), can describe also the evolution of surfaces in time. The interaction between differential geometry of surfaces and NDE has been studied since the 19th centure. This relationship is based on the fact that most of the local properties of surfaces are expressed in terms of NDE. Since the famous sine-Gordon and Liouville equations, the interrelation between NDE of the classical differential geometry of surfaces and modern soliton equations has been studied by various points of view in numerous papers. In particular, the relationship between deformations of surfaces and integrable systems in 2+1 dimensions were studied by several authors [1-14].

The self-dual Yang-Mills equation (SDYME) is a famous example of NDE in four dimensions integrable by the inverse scattering method [16-17]. Ward conjectured that all integrable (1+1)-dimensional NDE may be obtained from SDYME by reduction [18] (see the book [19] and references therein). More recently, many soliton equations in 2+1 dimensions have been found as reductions of the SDYME [20-23].

In this paper we study the deformation of surfaces in the context of its connection with integrable systems in 2+1 and 3+1 dimensions. We conjecture that many integrable (2+1)-dimensional NDE can be obtained from the deformed or (2+1)-dimensional Gauss-Mainardi-Codazzi equation (dGMCE) describing the deformation (motion) of the surface, as exact particular cases. At the same time, integrable isotropic spin systems (SS) in 2+1 dimensions are exact reductions of the (2+1)-dimensional or in the other words, deformed Gauss-Weingarten equation (dGWE). This statement is presented as a conjecture. Also we show that the dGMCE is the exact reduction of two famous multidimensional integrable system, namely, the Yang-Mills-Higgs-Bogomolny equation (YMHBE) and the SDYME.

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## 2  Fundamental facts on the theory of surfaces

Let us consider a smooth surface in $R^3$ with local coordinates $x$ and $t$, where $\mathbf{r}(x,t)$ is a position vector. The first and second fundamental forms of this surface are given by

$$I = d\mathbf{r}^2 = Edx^2 + 2Fdxdt + Gdt^2, \qquad II = d\mathbf{r}\cdot\mathbf{n} = Ldx^2 + 2Mdxdt + Ndt^2 \tag{1}$$

where by defintion $E = \mathbf{r}_x^2$, $\quad F = \mathbf{r}_x\cdot\mathbf{r}_t$, $\quad G = \mathbf{r}_t^2$ and $L = \mathbf{r}_{xx}\cdot\mathbf{n}$, $\quad M = \mathbf{r}_{tx}\cdot\mathbf{n}$, $\quad N = \mathbf{r}_{tt}\cdot\mathbf{n}$. The unit normal vector $\mathbf{n}$ to the surface is given by $\mathbf{n} = \frac{\mathbf{r}_x\wedge\mathbf{r}_t}{|\mathbf{r}_x\wedge\mathbf{r}_t|}$. There exist the third fundamental form

$$III = d\mathbf{n}\cdot d\mathbf{n} = edx^2 + 2fdxdt + gdt^2. \tag{2}$$

This form, in contrast to $II$, does not depend on the choice of $\mathbf{n}$ and contains no new information, since it is expressible in terms of $I$ and $II$ as

$$III = 2H\cdot II - K\cdot I \tag{3}$$

where $K, H$ are the gaussian and mean curvatures, respectively. As is well known in surface theory, the Gauss-Weingarten equation (GWE) for surface can be written as

$$\mathbf{r}_{xx} = \Gamma_{11}^1\mathbf{r}_x + \Gamma_{11}^2\mathbf{r}_t + L\mathbf{n}, \quad \mathbf{r}_{xt} = \Gamma_{12}^1\mathbf{r}_x + \Gamma_{12}^2\mathbf{r}_t + M\mathbf{n}, \quad \mathbf{r}_{tt} = \Gamma_{22}^1\mathbf{r}_x + \Gamma_{22}^2\mathbf{r}_t + N\mathbf{n} \tag{4a}$$

$$\mathbf{n}_x = P_1^1\mathbf{r}_x + P_1^2\mathbf{r}_t, \qquad \mathbf{n}_t = P_2^1\mathbf{r}_x + P_2^2\mathbf{r}_t \tag{4b}$$

where

$$\Gamma_{11}^1 = \frac{GE_x - 2FF_x + FE_t}{2g}, \quad \Gamma_{11}^2 = \frac{2EF_x - EE_t - FE_x}{2g}, \quad \Gamma_{12}^1 = \frac{GE_t - FG_x}{2g} \tag{5a}$$

$$\Gamma_{12}^2 = \frac{EG_x - FE_t}{2g}, \quad \Gamma_{22}^1 = \frac{2GF_t - GG_x - FG_t}{2g}, \quad \Gamma_{22}^2 = \frac{EG_t - 2FF_t + FG_x}{2g}. \tag{5b}$$

$$P_1^1 = \frac{MF - LG}{g}, \quad P_1^2 = \frac{LF - ME}{g}, \quad P_2^1 = \frac{NF - MG}{g}, \quad P_2^2 = \frac{MF - NE}{g}. \tag{5c}$$

Here $g = EG - F^2$. Now we introduce the orthogonal basis as $\mathbf{e}_1 = \frac{\mathbf{r}_x}{\sqrt{E}}$, $\quad \mathbf{e}_2 = \mathbf{n}$, $\quad \mathbf{e}_3 = \mathbf{e}_1\wedge\mathbf{e}_2$. Hence $\mathbf{r}_t = \frac{F}{\sqrt{E}}\mathbf{e}_1 - \sqrt{\frac{g}{E}}\mathbf{e}_3$. Then the GWE (4) takes the form

$$\begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{pmatrix}_x = A\begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{pmatrix}, \qquad \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{pmatrix}_t = B\begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{pmatrix} \tag{6}$$

where

$$A = \begin{pmatrix} 0 & k & -\sigma \\ -k & 0 & \tau \\ \sigma & -\tau & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix} \tag{7}$$

and $k_g = k = \frac{L}{\sqrt{E}}$, $\quad \tau_g = \tau = -\sqrt{\frac{g}{E}}P_1^2$, $\quad k_n = \sigma = \frac{\sqrt{g}}{E}\Gamma_{11}^2$ and $\omega_1 = -\sqrt{\frac{g}{E}}P_2^2$, $\quad \omega_2 = \frac{\sqrt{g}}{E}\Gamma_{12}^2$, $\quad \omega_3 = \frac{M}{\sqrt{E}}$. Here $k_n, k_g, \tau_g$ are called the normal curvature, geodesic curvature and geodesic torsion, respectively. In the case $\sigma = 0$ the first equation of the GWE (6) coincides in fact with the Frenet equation for the curves. So, all that we are doing in the next is true for the motion (deformation) of curves when $\sigma = 0$.

The compatibilty condition for the GWE (6) gives the Gauss-Mainardi-Codazzi equation (GMCE) as

$$A_t - C_x + [A, C] = 0 \tag{8}$$

or in elements

$$k_t = \omega_{3x} + \tau\omega_2 - \sigma\omega_1, \quad \tau_t = \omega_{1x} + \sigma\omega_3 - k\omega_2, \quad \sigma_t = \omega_{2x} + k\omega_1 - \tau\omega_3. \tag{9}$$

We can reformulate the linear system (6) in $2 \times 2$ matrix form as $\phi_x = U\phi, \quad \phi_t = V\phi$, where

$$U = \frac{1}{2i}\begin{pmatrix} \tau & k + i\sigma \\ k - i\sigma & -\tau \end{pmatrix}, \quad V = \frac{1}{2i}\begin{pmatrix} \omega_1 & \omega_3 + i\omega_2 \\ \omega_3 - i\omega_2 & -\omega_1 \end{pmatrix}.$$

## 3   Deformation of surfaces

Now we would like to consider the deformation of the surface with respect to $y$. We postulate that such deformation or motion of the surface is governed by the system [25]

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_x = A\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}, \quad \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_y = B\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}, \quad \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_t = C\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \tag{10}$$

where

$$A = \begin{pmatrix} 0 & k & -\sigma \\ -k & 0 & \tau \\ \sigma & -\tau & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & \gamma_3 & -\gamma_2 \\ -\gamma_3 & 0 & \gamma_1 \\ \gamma_2 & -\gamma_1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix} \tag{11}$$

and $\gamma_j$ are real functions. The system (10) will be called the deformed or (2+1)-dimensional GWE (for short, dGWE). We remark that first and third equations of the system (10) are the equations (6) and $A, B$ coincide with formulas (7). The compatibility conditions of the dGWE (10) gives the deformed or (2+1)-dimensional GMCE (shortly, dGMCE) of the form [25]

$$A_t - C_x + [A, C] = 0 \tag{12a}$$

$$A_y - B_x + [A, B] = 0 \tag{12b}$$

$$B_t - C_y + [B, C] = 0. \tag{12c}$$

As we see, equation (12a) is in fact the GMCE (8). This fact explains why we call (12) the deformed or (2+1)-dimensional GMCE. The linear problem (Lax representation) associated with the system (12) can be written as

$$\Psi_z = \lambda^2 \Psi_{\bar{z}} + (F^- - \lambda^2 F^+)\Psi, \quad \Psi_t = -i\lambda\Psi_{\bar{z}} + (C + i\lambda F^+)\Psi \tag{13}$$

where $F^{\pm} = A \pm iB$ and $z = \frac{1}{2}(x + iy), \quad \bar{z} = \frac{1}{2}(x - iy)$. So we can confirm that the dGMCE (12) is a candidate to be integrable in the sense that for it there exist the Lax representation with the spectral parameter (13). Higher hierarchy of the (2+1)-dimensional GMCE (12) can be obtained as the compatibility condition of the linear system [25]

$$\Psi_z = \lambda^2 \Psi_{\bar{z}} + (F^- - \lambda^2 F^+)\Psi, \quad \Psi_t = -i\lambda^n \Psi_{\bar{z}} + \sum_{j=0}^{m} \lambda^j F_j \Psi. \tag{14}$$

## 4   Deformation of surfaces induced by (2+1)-dimensional integrable systems

In this section we would like to attract an attention on some aspects of the relation between the deformation of surfaces and integrable systems in 2+1 dimensions. Now we make some conjectures.

## 4.1 Integrable systems in 2+1 dimensions and the deformed GMCE

**Conjecture 1.** *Many (maybe all) integrable systems in 2+1 dimensions are particular reductions of equations (12)* [25].

The well known (2+1)-dimensional integrable systems such as the KP and mKP equations, the Davey-Stewartson (DS) equation and so on, can be obtained from the dGMCE (12) as some reductions. For instance, the DS-II equation can be obtained from the (2+1)-dimensional GMCE (12) as

$$A = \sqrt{2}i\lambda\sigma_3 + \frac{1}{\sqrt{2}}\bar{q}\sigma^+ + \frac{1}{\sqrt{2}}q\sigma^-, \qquad B = -\frac{i\lambda}{\sqrt{2}}\sigma_3 + \frac{1}{\sqrt{2}}\bar{q}\sigma^+ + \frac{1}{\sqrt{2}}q\sigma^- \qquad (15a)$$

$$C = -\frac{i}{2}(|q|^2 + \phi_y + 3\lambda^2)\sigma_3 - 3\lambda\bar{q}\sigma^+ - 3\lambda q\sigma^-, \qquad \sigma^\pm = \sigma_1 \pm i\sigma_2 \qquad (15b)$$

where using isomorfism $so(3) \cong su(2)$, the matrices $A, B, C$ can be written in $2 \times 2$ form. Substituting (15) into the system (12) after some algebra we get the DS-II equation [24]

$$iq_t + \frac{1}{2}(q_{xx} - q_{yy}) - (|q|^2 + \phi_y)q = 0, \qquad \phi_{xx} + \phi_{yy} + 2(|q|^2)_y = 0. \qquad (16)$$

## 4.2 Integrable SS in 2+1 dimensions and the dGWE

The conjecture 1 is true also for integrable SS in 2+1 dimensions. But for isotropic subclass of such SS, the following conjecture takes places .

**Conjecture 2.** *Many (and maybe all) integrable isotropic SS in 2+1 dimensions are particular reductions of equations (10)* [25].

As an example, we consider the isotropic Myrzakulov I (M-I) equation

$$\mathbf{S}_t = (\mathbf{S} \wedge \mathbf{S}_y + u\mathbf{S})_x \qquad (17a)$$

$$u_x = -\mathbf{S} \cdot (\mathbf{S}_x \wedge \mathbf{S}_y) \qquad (17b)$$

which is integrable. In this case we take the identification $\mathbf{e}_1 = \mathbf{S}$, where $\mathbf{S}$ is the solution of the M-I equation (17) and $k^2 + \sigma^2 = \mathbf{S}_x^2$. Then the M-I equation (17) becomes

$$\mathbf{e}_{1t} = (\mathbf{e}_1 \wedge \mathbf{e}_{1y} + u\mathbf{e}_1)_x, \quad u_x = -\mathbf{e}_1 \cdot (\mathbf{e}_{1x} \wedge \mathbf{e}_{1y}). \qquad (18)$$

Now let us assume

$$\tau = f_x, \qquad \gamma_1 = f_y + u, \qquad \omega_1 = f_t + \partial_x^{-1}(\sigma\omega_3 - k\omega_2)$$

$$\omega_2 = -\gamma_{3x} - \gamma_2\tau + u\sigma, \quad \omega_3 = \gamma_{2x} - \gamma_3\tau + uk \qquad (19)$$

where $f(x, y, t, \lambda)$ is a real function. Taking into account formulas (19) and after eliminating the vectors $\mathbf{e}_2$ and $\mathbf{e}_3$, the system (10) takes the form (18). This means that the M-I equation (17) is the particular exact reduction of the (2+1)-dimensional GWE (10) with the choice (19). Similarly, we can show that the other isotropic SS in 2+1 dimensions are the exact reductions of the system (10) so that the conjecture 2 is true at least for existing known integrable isotropic (2+1)-dimensional SS [25].

### 4.3 Integrable SS as exact reductions of the M-0 equation

Now let us consider the (2+1)-dimensional isotropic Myrzakulov 0 (M-0) equation (about our notations, see, i.e., [13-15])

$$\mathbf{e}_{1t} = \omega_3 \mathbf{e}_2 - \omega_2 \mathbf{e}_3, \qquad \tau_y - \omega_{1x} = \mathbf{e}_1 \cdot (\mathbf{e}_{1x} \wedge \mathbf{e}_{1y}) \tag{20}$$

which sometimes we write in terms of $\mathbf{S}$ as

$$\mathbf{S}_t = \theta_1 \mathbf{S}_x + \theta_2 \mathbf{S}_y, \qquad \tau_y - \omega_{1x} = \mathbf{S} \cdot (\mathbf{S}_x \wedge \mathbf{S}_y) \tag{21}$$

where $\theta_j$ are some real functions. The following conjecture takes places.

**Conjecture 3.** *Many (and maybe all) integrable isotropic SS in 2+1 dimensions are particular reductions of the (2+1)-dimensional isotropic M-0 equation (20)-(21)* [25].

For example, the M-I equation (17) is the particular case of (21) as

$$\theta_1 = \frac{\omega_3 \gamma_2 - \omega_2 \gamma_3}{k \gamma_2 - \sigma \gamma_3}, \quad \theta_2 = \frac{\omega_2 k - \omega_3 \sigma}{k \gamma_2 - \sigma \gamma_3}. \tag{22}$$

## 5 The (2+1)-dimensional GCME as exact reduction of the Yang-Mills-Higgs-Bogomolny equation

One of the most interesting and important integrable equations in 2+1 dimensions is the following Yang-Mills-Higgs-Bogomolny equation (YMHBE) [19]

$$\Phi_y + [\Phi, B] + C_x - A_t + [C, A]] = 0 \tag{23a}$$

$$\Phi_t + [\Phi, C] + A_y - B_x + [A, B] = 0 \tag{23b}$$

$$\Phi_x + [\Phi, A] + B_t - C_y + [B, C] = 0. \tag{23c}$$

The important observation is that the dGMCE (12) is the particular case of the YMHBE (23). In fact, if in the YMHBE we put $\Phi = 0$ then the YMHBE (23) becomes the dGMCE (12). So we can suggest that the dGMCE is a candidate to be integrable as the exact reduction of the integrable equation (23).

## 6 The (2+1)-dimensional GCME as exact reduction of the Self-Dual Yang-Mills equation

Now we study the relationship between the dGMCE (12) and the SDYM equation. The SDYME reads as

$$F_{\mu\nu} = {}^* F_{\mu\nu} \tag{24}$$

where $^*$ is the Hodge star operator and the Yang-Mills field defined as $F_{\mu\nu} = \frac{\partial A_\nu}{x_\mu} - \frac{\partial A_\mu}{x_\nu} - [A_\mu, A_\nu]$. Let $x_\alpha = z + it$, $x_{\bar\alpha} = z - it$, $x_\beta = x + iy$, $x_{\bar\beta} = x - iy$ be the null-coordinates in Euclidean space for which the metric has the form $ds^2 = dx_\alpha dx_{\bar\alpha} + dx_\beta dx_{\bar\beta}$. Now the SDYME takes the form [16-17,19]

$$F_{\alpha\beta} = 0, \quad F_{\bar\alpha\bar\beta} = 0, \quad F_{\alpha\bar\alpha} + F_{\beta\bar\beta} = 0 \tag{25}$$

where $A_\alpha = A_z + iA_t$, $A_{\bar\alpha} = A_z - iA_t$, $A_\beta = A_x + iA_y$, $A_{\bar\beta} = A_x + iA_y$. The associated linear system is [19]

$$(\partial_\alpha + \lambda \partial_{\bar\beta})\Psi = (A_\alpha + \lambda A_{\bar\beta})\Psi, \qquad (\partial_\beta - \lambda \partial_{\bar\alpha})\Psi = (A_\beta - \lambda A_{\bar\alpha})\Psi \tag{26}$$

where $\lambda$ is the spectral parameter and

$$\frac{\partial}{\partial x_\alpha} = \frac{\partial}{\partial z} - i\frac{\partial}{\partial t}, \quad \frac{\partial}{\partial x_{\bar{\alpha}}} = \frac{\partial}{\partial z} + i\frac{\partial}{\partial t}, \quad \frac{\partial}{\partial x_\beta} = \frac{\partial}{\partial x} - i\frac{\partial}{\partial y}, \quad \frac{\partial}{\partial x_\alpha} = \frac{\partial}{\partial x} + i\frac{\partial}{\partial t}. \quad (27)$$

Our second observation: the dGMCE (12) is the particular reduction of the SDYME (25). In fact, we consider the following reduction of the SDYME

$$A_\alpha = -iC, \quad A_{\bar{\alpha}} = iC, \quad A_\beta = A - iB, \quad A_{\bar{\beta}} = A + iB \quad (28)$$

and assume that $A, \quad B, \quad C$ are independent of $z$. In this case, from the SDYME (25) we obtain the (2+1)-dimensional GMCE (12) in Euclidean coordinates.

# 7  Conclusion

In this paper, we have considered some deformations or in other terminology, motions of surfaces. We have shown that the corresponding dGMCE is integrable in the sense that the associated linear problem (Lax representation) exists with the spectral parameter. We conjectured that many (maybe all) integrable systems in 2+1 dimensions are some reductions of the dGMCE. In particular, as example we proved how the DS-II equation can be obtained from the dGMCE. Although, all known integrable (2+1)-dimensional isotropic SS can be obtained from the dGMCE, we have conjectured that such SS can be obtained from the dGWE as exact reductions. Finally, we proved that the dGMCE is the particular case of two famous integrable systems namely the YMHBE and SDYME. It goes in favour of integrability of the dGMCE.

# 8  Acknowledgments

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